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International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 3 – DISCRETE MATHEMATICS

Tuesday 21 May 2013 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 15]*

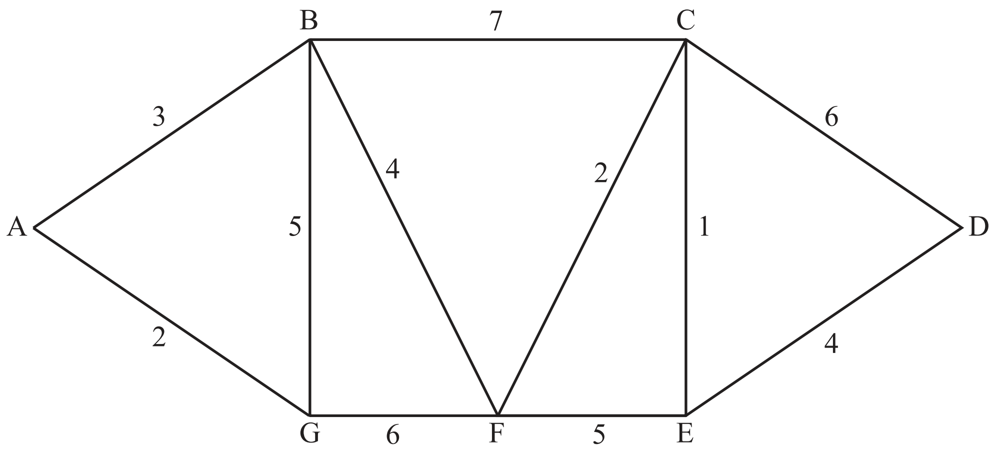
(a) Using the Euclidean algorithm, show that $\gcd(99, 332) = 1$. *[4 marks]*

(b) (i) Find the general solution to the diophantine equation $332x - 99y = 1$.

(ii) Hence, or otherwise, find the smallest positive integer satisfying the congruence $17z \equiv 1 \pmod{57}$. *[11 marks]*

2. *[Maximum mark: 12]*

The diagram shows a weighted graph with vertices A, B, C, D, E, F, G. The weight of each edge is marked on the diagram.



(a) (i) Explain briefly why the graph contains an Eulerian trail but not an Eulerian circuit.

(ii) Write down an Eulerian trail. *[4 marks]*

(b) (i) Use Dijkstra's algorithm to find the path of minimum total weight joining A to D.

(ii) State the minimum total weight. *[8 marks]*

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3. *[Maximum mark: 10]*

When numbers are written in base n, $33^2 = 1331$.

(a) By writing down an appropriate polynomial equation, determine the value of n. *[4 marks]*

(b) Rewrite the above equation with numbers in base 7. *[6 marks]*

4. *[Maximum mark: 15]*

The graph G has the following adjacency matrix.

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(a) Determine the number of walks of length five beginning and ending at E. *[3 marks]*

(b) Show that G and its complement G' have the same number of edges. *[3 marks]*

(c) (i) Writing the vertices in the order B, D, A, C, E, determine the adjacency matrix of G'.

(ii) Deduce that G and G' are isomorphic. *[5 marks]*

(d) The graph H has 6 vertices. Show that H and H', the complement of H, cannot be isomorphic. *[4 marks]*

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5. *[Maximum mark: 8]*

The positive integer p is an odd prime number.

(a) Show that $\sum_{k=1}^{p} k^p \equiv 0 \pmod{p}$. *[4 marks]*

(b) Given that $\sum_{k=1}^{p} k^{p-1} \equiv n \pmod{p}$ where $0 \le n \le p-1$, find the value of n. *[4 marks]*

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